Joel Pokorny, Co-authors of this paper are Vivianne C. Smith, Stephen A. Burns, Ann Elsner and Qasim Zaidi. Drs. Burns and Elsner are currently at the Eye and Ear Hospital, 230 Lothrop St., Pittsburgh, PA 15213, Chicago (IL):

Modeling Blue-Yellow Opponency**

Linear models of yellow-blue opponency are easily constructed but data suggest non-linearities. Some requirements which a non-linear model should fulfill are discussed and published models are evaluated. It is noted that it is inappropriate to model the valence curve with a non-linear combination of pigment sensitivities.

Il n'est pas difficile de construire un modèle linéaire de l'opposition jaune-bleu, mais les résultats souhaitent un modèle non-linéaire. On discute quelques exigences d'un tel modèle qui doivent être accomplies et on regarde des modèles déjà publiés. On a trouvé, qu'il n'est pas approprié de présenter la courbe des valences par une combination non-linéaire des sensibilités des pigments.

Lineare Modelle für den Gelb-Blau-Gegensatz lassen sich leicht konstruieren, aber die Versuchsergebnisse legen eine Nicht-Linearität nahe. Es werden hier einige Erfordernisse besprochen, die ein nicht-lineares Modell erfüllen muß, und schon veröffentlichte Modelle werden kritisch untersucht. Dabei zeigt sich, daß es unangebracht ist, die Valenzkurven durch eine nicht-lineare Kombination der Pigmentempfindlichkeiten darzustellen.

Reprinted from Proc. 4th Int. Congr. Int.
Color Assoc. (AIC)
M. Richter, Ed. (Berlin, 1981)

The Author's address:

Prof. Joel Pokorny, Ph.D.
Eye Research Laboratories, University of Chicago
939 E. 57th St.
Chicago, IL 60637

**Supported in part by NIH USPH research grant R01 EY00901. Drs. Burns and Elsner were supported by NIH USPH training grant T32 EY007010.

Many modern color theories postulate that signals arising from cone excitation in three cone types are reorganized in the nervous system to two opponent color channels and one additive luminance channel. Some theories, those postulated for example by Hurvich and Jameson (1955), by Judd and Yonemura (1969) and by Guth, Massof and Benzschawel (1980) use linear modeling. The chromatic component is assumed to reflect a linear combination of cone sensitivities. For this paper we use estimates of the cone sensitivities (Smith and Pokorny, 1975) expressed in terms of a set of CIE (Judd, 1951) revised color matching functions:

$$\text{SWS:}\quad S_\lambda = \bar{z}_\lambda \tag{1}$$

$$\text{MWS:}\quad M_\lambda = -0.15514\bar{x}_\lambda + 0.45684\bar{y}_\lambda + 0.03286\bar{z}_\lambda \tag{2}$$

$$\text{LWS:}\quad L_\lambda = 0.15514\bar{x}_\lambda + 0.54314\bar{y}_\lambda - 0.03286\bar{z} \tag{3}$$

The blue-yellow opponent is then derived by linear subtraction of a "blueness" component given by the SWS curve with a "yellowness" component; for linear models $(y,b)_\lambda$ valence curves are directly expressed in equations involving CIE color matching functions. According to the precise theoretical standpoint, the "yellowness" component may be represented as a sum of MWS and LWS (equation 4, e.g. Hurvich and Jameson (1955); MWS (equation 5, e.g. Judd and Yonemura, 1969); or LWS (equation 6, e.g. Guth, Massof and Benzschawel, 1980). Since "white" is perceived by young observers for spectral distributions near equal energy (Priest, 1921; Hurvich and Jameson, 1951; Valberg, 1971, Burns et al, 1979), we constrain all linear models to have zero $(y,b)_\lambda$ valence at equal energy white ($\bar{x} = \bar{y} = \bar{z}$); to accomplish this, we adjust the coefficient of $\bar{z}$ so that the sum of coefficients is zero. The maximal heights of the valence curves are arbitrary and depend on the exact nature of the model. By convention, the blue valence is negative, For ease of comparison, in equations (4) - (6) constant multipliers are used so that the blue-valence is 0.4089 at 470 nm.

$$(y,b)_\lambda = 0.4\,(\bar{y}_\lambda - \bar{z}_\lambda) \tag{4}$$

$$(y,b)_\lambda = 1.27\,(-.15514\bar{x}_\lambda + .45684\bar{y}_\lambda - .3017\bar{z}_\lambda) \tag{5}$$

$$(y,b)_\lambda = 0.583\,(.15514\bar{x}_\lambda + 0.54314\bar{y} - 0.69828\bar{z}_\lambda) \tag{6}$$

When the output of the (y,b) channel is set to zero, a set of null coordinates for the yellow-blue opponent may be derived and plotted in the CIE (Judd, 1951) x, y diagram. For this purpose, equations (4) - (6) may be rewritten as functions of x and y:

$$y = 0.5 - .5x \tag{4a}$$

$$y = .3977 - .1932x \tag{5a}$$

$$y = .5625 - .6875x \tag{6a}$$

In the CIE x, y diagram, linear models predict a line passing through the equal energy white, intersecting the spectrum locus at or near 500 nm, the locus of equilibrium green for each theory. Each line also intersects the locus of extra-spectral purples predicting that some short wavelength light must be added to long wavelength lights to give the appearance of equilibrium red. According to linear models, equilibrium red is complementary to equilibrium green.

Linear models have two important properties: First the valence of a mixture of two lights equals the sums of their valences:

$$(y,b)_{\lambda_1 + \lambda_2} = (y,b)_{\lambda_1} + (y,b)_{\lambda_2} \qquad (7)$$

Second, the null coordinates are luminance invariant. In a linear model, it is usual to give valence curves for an equal energy spectrum. A change in radiance level changes the heights of the valence curves but does not change the position of the null coordinates in the CIE diagram. For example, suppose there is a ten-fold increase in radiance. the $(y,b)_\lambda$ valence curves (equations 4-6) are multiplied by 10; this factor of course drops out in (4a) - (6a). It is irrelevant whether the x,y coordinates are calculated for equal luminance or equal radiance spectra, the null coordinates will be identical. It follows from these properties that $|(y,b)_{\lambda_1}|$ units of λ_2 will cancel $|(y,b)_{\lambda_2}|$ units of λ_1 if the valences for λ_1 and λ_2 are of opposite sign; therefore a linear chromatic valence function is independent of the choice of cancellation of light except for changes of unit. This discussion applies equally to chromatic response functions expressed in terms of cone sensitivities. Both properties will also hold for those non-linear functions which can be treated as linear at equilibrium i.e. for the set of null coordinates.

Although linear models are attractive for their simplicity there are accumulating data indicating that the blue-yellow opponent is not linear. For example (1), unique red is not complementary to unique green (Dimmick and Hubbard, 1939). (2), Valberg (1971) showed that the loci of null points for blue-yellow perception are not collinear. We (Burns et al, 1979) have extensive data confirming Valberg (1971) for a larger region of chromaticity space. In these two studies, null coordinates for blue-yellow perception were determined for various color mixtures and plotted in the CIE diagram. There is a kink near equal energy white and data points connect from unique green to the equal energy white and then turn toward the red corner of the diagram. (3), Ikeda and Ayama (1980) have observed additivity failures of the yellowness component of mixtures or orange and yellow-green lights, a result which implies that two cone types contribute to yellowness and the responses of those cone types do not add linearly to produce the yellow section. (4), Larimer, Krantz and Cicerone (1975) have presented evidence that yellow-blue equilibria are not luminance invariant for a 100-fold change in luminance. They measured changes in unique green and unique red. They derived the following non-linear model from their data:

$$\phi_\lambda = -S_\lambda + k_1 M_\lambda \mp k_2 |[L_\lambda - M_\lambda]|^n \qquad (8)$$

In equation (8), S_λ, and M_λ, and L_λ are pigment sensitivities such as those given in equations (1) - (3); k_1 and k_2 are weighting constants; and n allows compression of the L-M cone difference signal.

The sign of the non-linear portion is positive when $L_\lambda > M_\lambda$ (above 500 nm for the normalization they used) and negative otherwise. Larimer, Krantz and Cicerone (1975) do not explicitly associate equation (8) with a valence, but they do imply that positive values of ϕ_λ will be associated with "yellow" percepts and negative values of ϕ_λ with "blue" percepts. (5), Werner and Wooten (1979) measured hue cancellation curves for an equal luminance spectrum, referring the data to the conventional equal radiance form. They then modeled the equal radiance valence curves, using an equation of the form:

$$(y,b)_\lambda = -k_1 S_\lambda + |[k_3 L_\lambda - k_2 M_\lambda|^n \qquad (9)$$

In considering methods of deriving non-linear models, we are faced with certain restrictions. In a cancellation experiment, the relative heights of the y and b lobes are determined by postulating that when two spectral stimuli, λ_1 and λ_2, are mixed to give an equilibrium percept (that is neither yellow nor blue) then the "yellowness" E_{λ_1} $(y,b)_{\lambda_1}$ of λ_1 is equal to the "blueness" E_{λ_2} $(y,b)_{\lambda_2}$ of λ_2. The amount of "yellowness" at λ_1 and "blueness" at λ_2 needed to cancel each other can be used as units of the "blueness" and "yellowness" This postulate depends on the assumption that the set of equilibria is closed under additive mixture and for radiance changes (Krantz, 1975). It is inappropriate to fit a non-linear model to a valence curve because the non-linearity contradicts this postulate underlying the normalization of heights. Therefore, two lights which will cancel in the proportions predicted by the empirical valence curve, will not add to equilibrium in the non-linear model which is fit to the valence curve.

The use of exponents greater than unity leads to conflicts with data when radiance change is introduced. Consider a ten-fold increase in radiance in the Werner & Wooten model, where the empirically fitted exponent, n, varied from 3.04-3.90. The yellowness component, $|10(L_\lambda - M_\lambda)|^3$, will grow one-hundred fold relative to the "blueness" component, $10S_\lambda$. Similarly with a ten-fold decrease in radiance the yellowness component will decrease 100-fold relative to the blueness component. Equilibrium green would have a 70 nm range and the achromatic point would change its chromaticity coordinates from near the usual spectral yellow to near the usual spectral blue.

The Larimer, Krantz & Cicerone model also has shortcomings since it was optimized for a restricted data set; if values of the Vos and Walraven (1970) spectral sensitivity functions are substituted in the model, two out of four observers show a "yellow" percept at very short wavelengths. Their equations do not produce chromaticity coordinates which intersect the "white" region of the diagram.

Non-linearities can also be introduced in other ways than expansion or compression. For example, consider the possibility that the blue-yellow opponent signal is affected by the red-green signal according to equation 10:

$$(y,b)_\lambda = .4\,(L_\lambda + M_\lambda - S_\lambda) - k\,|(L_\lambda - 1.99M_\lambda)| \qquad (10)$$

where S_λ, M_λ, L_λ are from equations 1-3, and k is a constant. The first term gives a conventional linear $(y,b)_\lambda$ channel identical to equation 4; the second term gives a linear $(r,g)_\lambda$ opponent similar to that defined by Guth, Massof and Benzschawel (1980). The effect of

equation 10 is that the absolute value of the $(r,g)_\lambda$ opponent is subtracted from the (y,b) opponent. This type of equation will give null coordinates that pass through equal energy white and that are not affected by changes in radiance. A luminance dependent or observer-dependent function may be obtained by varying k. An increase in k from 0.1 to 0.5 results in a 4 nm shift to longer wavelengths for unique green, while the yellowness component of long wavelength reds is reduced. Increasing k with luminance increase yields predictions which are consistent with data reported by Larimer, Krantz, and Cicerone (1975) that at higher luminance, less blue cancelling stimulus is necessary to cancel the yellowness content of long wavelength lights. The absolute value function will not be linear for all mixtures of lights. The non-linearity results from the inequality:

$$|(r,g)_{\lambda_1}| + |(r,g)_{\lambda_2}| \geq |(r,g)_{\lambda_1} + (r,g)_{\lambda_2}| \qquad (11)$$

Non-linearity occurs for mixture wavelengths of opposite red-green valence (i.e. $\lambda_1 > 570$ nm, $\lambda_2 < 570$ nm). The mixture color will appear yellower than predicted by a linear model. This type of finding was noted by Ikeda and Ayama (1980). The absolute value function provides a restricted type of non-linearity and maintains linear properties for many other conditions.

In summary, we have pointed out that many theorists have concluded that the (y,b) chromatic valence is non-linear. Some non-linear models using exponents have been proposed and optimized on restricted data sets. However, at least one model's derivation violates basic assumptions concerning the nature of equal energy valence curves. Other types of non-linearity may avoid some difficulties associated with the use of exponents. As an example we have used an absolute value function to introduce a non-linearity, but other types of non-linearity could presumably be used. The consequences of the non-linearity must be reviewed both for its effect on the chromaticity coordinates and for its effect under scalar multiplication and mixture additivity. We emphasize that non-linear models may be expressed in chromaticity space but it is inappropriate to model a valence curve using non-linear combinations of pigment sensitivities.

Burns, S.A., Elsner, A.E., Pokorny, J., and Smith, V.C.
Null coordinates for the opponent color channels.
J. Opt. Soc. Am. 69: 1406A, 1979

Dimmick, F.L. and Hubbard, M.A.
The spectral components of psychologically unique red.
Am. J. Psychol. 52: 348-353, 1939

Guth, S.L. Massof, R.W. and Benzschawel, J.
Vector model for normal and dichromatic color vision.
J. Opt. Soc. Am. 70: 197-212, 1980

Hurvich, L.M. and Jameson, D.
A psychophysical study of white. I. Neutral adaptation
J. Opt. Soc. Am. 41: 521-527, 1951

Hurvich, L.M., Jameson, D.
Some quantitative aspects of an opponent-colors theory. II. Brightness, saturation, and hue in normal and dichromatic vision.
J. Opt. Soc. Am. 45: 616, 1955

Ikeda, M. and Ayama, M.
Additivity of chromatic valence.
Vision Res. 20: 995-999, 1980

Judd, D.B.
Colormietry and artificial daylight, in Technical Committee No. 7 Report of Secretariat.
United States Commission, International Commission on Illumination, Twelfth Session, Stockholm, Sweden 1951a, pp 1-60

Judd, D.B. and Yonemura, G.T.
CIE 1960 UCS diagram and the Müller theory of color vision,
Proc. Int. Color Associations, Stockholm, Sweden
1969, 266-274 (Musterschmidt, Gottingen, Germany, 1970)

Krantz, D.L.
Color measurement and color theory: II. Opponent-colors theory.
J. Math. Psychol. 12: 304-327, 1975

Larimer, J., Krantz, D.H., Cicerone, C.M.
Opponent process additivity - II. Yellow/blue equilibria and non-linear models.
Vision res. 15: 723-731, 1975

Priest, I.G.
The spectral distribution of energy required to evoke the gray sensation.
J. Opt. Soc. Am. 5: 205-209, 1921

Smith, V.C., Pokorny, J.
Spectral sensitivity of the foveal cone photopigments between 400 and 500 nm.
Vision Res. 15: 161-171, 1975

Valberg, A.
A method for the precise determination of achromatic colors including white.
Vision Res. 11: 157-160, 1971

Vos, J.J., Walraven, P.L.
On the derivation of the foveal receptor primaries.
Vision Res. 11: 799-818, 1971

Werner, J.S. and Wooten, B.R.,
Opponent chromatic mechanisms: Relation to photopigments and hue naming.
J. Opt. Soc. Am. 69: 422-434, 1979